REPORT RESUMES

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THE TEACHER, PROGRAMED MATERIALS, AND INSTRUCTIONAL INTERACTION, SUPPLEMENTAL MANUAL.
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THIS IS A SOURCE MANUAL TO ASSIST TEACHERS OF PROGRAMED ALGEBRA IN PROVIDING FOR INDIVIDUAL DIFFERENCES AMONG STUDENTS. THE MATERIAL WAS INTENDED FOR ASSIGNMENT TO SELECTED STUDENTS WHO REQUIRE ADDITIONAL MATERIALS TO SATISFY VARIOUS ABILITIES AND TALENTS. IN ADDITION TO REFERENCE LISTS ON FILMS AND FILMSTRIPS, PRINTED BIBLIOGRAPHIES WERE PREPARED ON POPULAR EXPOSITIONS, HISTORY, BIOGRAPHY, TRADITIONAL MATHEMATICS, MODERN MATHEMATICS, SPECIAL TOPICS, MATHEMATICAL RECREATIONS, TEACHING AIDS, PROGRAMED INSTRUCTION, AND PROGRAMED ALGEBRA. OTHER REPORTS ON THIS PROJECT WERE ASSIGNED ACCESSION NUMBERS ED 003 197, ED 003 198, AND ED 003 238. (JM)

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SUPPLEMENTAL MANUAL

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SUPPLEMENTAL MANUAL

INTRODUCTION

This manual has been prepared for the express purpose of assisting the teacher in providing for individual differences among the students. The two general areas covered are drill problems and enrichment activities. The material found in this manual is provided merely as a suggestion to indicate the type of work which might be considered and is by no means complete within itself. The teacher is encouraged to use this material and to revise the suggestions by additions or deletions depending on the demands of his particular situation. The use of both the drill and enrichment materials will enable the teacher to provide a method by which the course may be supplemented in order to meet the particular needs of the student. This material is not intended to be assigned to the group at various intervals during the programed course but rather to selected students who require additional materials to satisfy various abilities and talents. One of the primary functions of the teacher will be to identify these particular needs and then to differentiate the course to meet these needs.

SUPPLEMENTARY DRILL

The primary purpose of the drill material is to fixate certain principles and concepts. While the actual method in which the materials will be employed is left to the discretion of the teacher, there are two methods which have merit. The first is to have the drill materials available to all students who, upon completing a unit of study, feel a need for further practice before being tested on the unit. In this instance much of the responsibility would rest with the student. The second method would be to use this material after providing remedial instruction to a student who has demonstrated on a unit test an insufficient understanding of a specific concept. Both these methods may be used concurrently in order to provide a more comprehensive understanding of the materials. It is necessary that each student demonstrate a certain degree of proficiency before being allowed to continue on to the next unit of study. Again, it should be emphasized that the problems represent only a suggestion as to the type of drill to be provided for the student and definitely do not represent a complete set of materials. The teacher may further supplement the program by additional drill materials to insure competency on the part of the student.

SUPPLEMENTARY ENRICHMENT

Suggestions for enrichment activities have been listed primarily for those students who have demonstrated a high degree of proficiency in working the programed materials. The topics and projects found in this section of the manual cover a wide spectrum of interest and are of varying degrees of difficulty. Some of these topics can be covered in a few days while others will require almost a complete semester's work. These materials are not designed to provide a means of additional work for the student, but rather as a method by which the opportunity is afforded the student to explore topics of interest. Included in the suggested enrichment work are biographies of famous mathematicians, selected topics of study, and creative activities. Each of the listed items in the selected topics is followed by a series of sub-topics and numbers that correspond to specific references to be found in the bibliography. Other topics and references may be selected either by the student or the teacher. In teaching conditions I, II, and III, the enrichment work would be given concurrently with the work in the program itself. In Conditions IV and V the enrichment work is reserved until the program work is first mastered and then the remainder of the course time may be assigned to enrichment materials. Any enrichment project which is performed by the student should be presented to the class wherever applicable.

PROGRAMED MATERIALS

An additional type of study may be undertaken by the student through the use of other programed materials. These present the opportunity for a student to further increase his knowledge and background in mathematics. A prospectus of these courses will provide a description of the course content and other pertinent information concerning the materials. In the situation where the student covers a complete course of additional work, a record should be made of this on his permanent record card and if possible a mark should be given to designate the degree of proficiency demonstrated by the student.

FILM AND FILMSTRIPS

The use of films and filmstrips can offer effectively improved mathematical instruction if the lesson is properly structured. The teacher may use films to present additional topics to the class or to provide variance in the method of instruction. Also, films and filmstrips could be used to aid an individual student in the understanding of a particular topic of enrichment work. The majority of these films can be obtained through the various film rental libraries or purchased at a nominal cost. A word of caution should be added to insure that the proper use of the film for instructional purpose is obtained. The films are only as effective as the structure in which they are employed and sufficient pre-planning should be carried out before they are utilized.

BIBLIOGRAPHY

The bibliography is designed to serve two purposes. One is to present a list of reference information in conjunction with the student enrichment material and the second purpose is to provide a reference source for the teachers. These books and articles contain many topics which are pertinent to the study of mathematics and are suggested as a means by which a teacher may further expand his knowledge in a particular subject.

UNIT 1

Supplementary Drill **Frames 1-502**

A. ORDER OF OPERATION:

		ANSWER			ANSWER
1.	$8 - 2 \div 2$	7	11.	$6(4) \div 3 + 8$	16
2.	$10 + 2(6)$	22	12.	$4 \div 2 + 3(2)$	8
3.	$3 - 2 - 1$	0	13.	$14 - 2(6) \div 1$	2
4.	$24 \div 4 - 2(3)$	0	14.	$8(8) \div 16 + 6$	10
5.	$6 + 2(2) \div 4 - 5$	2	15.	$18 \div 2(3) - 2$	1
6.	$8 \div 2 \div 2$	2	16.	$7(2) - 2 + 3(1)$	15
7.	$3(6) \div 9 - 1$	1	17.	$6 \div 2 \div 3 \cdot (7) + 1$	8
8.	$7 \div 7 + 1(6)$	7	18.	$8 \div .4 - 6(.1)$	19.4
9.	$10 + 5 \div 5$	11	19.	$2.5 + 6.2(.2)$	3.74
10.	$6(2) \div 3 + 1$	5	20.	$3\frac{1}{2} \div \frac{1}{2} + 3(2)$	13

B. ALGEBRAIC EXPRESSIONS (let x represent the number)

		ANSWER
1.	The sum of a and b	$a + b$
2.	x added to 10	$10 + x$
3.	Six less then y	$y - 6$
4.	Eight plus 6	$8 + 6$
5.	The product of 7 and x	$7x$
6.	Four times x increased by 6	$4x + 6$
7.	Three times a number increased by 6	$3x + 6$
8.	Seven times a number minus one-half the number	$7x - \frac{1}{2}x$
9.	x more then y	$y + x$
10.	Two x divided by y minus x times y	$(2x/y) - xy$
11.	The quotient of x divided by 25	$x/25$
12.	The sum of a number and 12	$a + 12$

13.	Ten greater than the number	$x + 12$
14.	Half of a number	$x/2$
15.	The sum of twice the number and a	$2x + a$

In the following problems let the first number equal a, the second number equal b, the third equal c, and fourth equal d.

C. Express the following statements by an algebraic expression:

1.	The sum of the four numbers.	$a + b + c + d$
2.	The product of the first and the fourth.	ad
3.	The second divided by the fourth.	b/d
4.	The product of the second and third number less three times the first.	$bc - 3a$
5.	Four times the first number divided by three.	$4a/3$
6.	Twice the first number decreased by five times the second.	$2a - 5b$
7.	The product of the first, second and fourth number minus three times the third.	$abd - 3c$
8.	The quotient of the second number divided by the first diminished by six.	$(b/a) -6$
9.	The quotient of three times the third number divided by the sum of the first and the fourth.	$\frac{3c}{a+d}$
10.	The difference of the quotient of the second number divided by the fourth and the quotient of the third number divided by the first.	$(\frac{b}{d}) - (\frac{c}{a})$

UNIT 2

Supplementary Drill

Frames 503-975

A. Prime numbers.

	ANSWERS
1. IDENTIFY WHICH OF THE FOLLOWING ARE PRIME NUMBERS: 2, 8, 9, 10, 17, 19, 21, 87, 93, 95, 97, 108, 276, 308, 309, 311, 477, 501.	2, 17, 19, 87, 97 311, 477, 501

WRITE THE PRIME FACTORS OF THE FOLLOWING WHERE POSSIBLE:

1. 12	2(2)(3)
2. 100	2(2)(5)(5)
3. 18	2(3)(3)
4. 8	2(2)(2)
5. 17	prime
6. 28	2(2)(7)
7. 48	(2)(2)(2)(2)(3)
8. 46	(23)(2)
9. 86	(43)(2)
10. 40	(2)(2)(2)(5)

B. WRITE IN EXPONENTIAL FORM:

1. xxxx	x^4
2. xxyy	x^2y^2
3. 3 3 xxx	3^2x^3
4. 7(7)	7^2
5. (5)(5)xxyyyy	$5^2x^2y^4$

C. FIND THE PRODUCT:

1. $x^3(x^4)(x^2)$	x^9
2. $x^2(y^3)(x^2)$	x^4y^3
3. $3x^2(4x^3)$	$12x^5$
4. $7x(x^3)$	$7x^4$

5.	$2ab(a^2b^3)(a^3b^5)$	$2a^6b^9$
6.	$4a^2b^2(3a)$	$12a^3b^2$
7.	$6r^2(rs)(r^2s^3)$	$6r^5s^4$
8.	$2r(3r)(4r)(r^3)$	$24r^6$
9.	$(xy)(xy)^2$	x^3y^3
10.	$6xyz(3x^2y)(2yx^2)$	$36x^3y^3z^3$
D.	EVALUATE THE FOLLOWING IF $a = 1$, $b = 2$, $c = 3$, $d = 5$, $e = 6$	
1.	$6b - 3$	9
2.	$ab + c$	5
3.	$abcd$	30
4.	$e/b - a$	2
5.	$3b - 2a$	4
6.	$d + e - b$	9
7.	$2a + 2b - e/c$	4
8.	$c - 2b$	32
9.	$ab + 3e^2$	110
10.	$a^2 + c^2 - b^2$	6
11.	$2a^3 + 2c^2 + ad$	25
12.	$d^2 - 2ab + c$	24
13.	$c^2 + 2ae + e$	27
14.	$2a^2b^2c^2$	72
15.	$a^2b^2c^2 - 3ab^2 + e^2$	60

UNIT 3

Supplementary Drill

Frames 976-1513

A. COMBINE TERMS:

		ANSWERS
1.	$2x + 3x + x$	$6x$
2.	$9x - 6x + 2x$	$5x$
3.	$3m + 2m - 5n$	0
4.	$9a + 2a - 3a + 2a$	$10a$
5.	$x + 3x - x$	$3x$
6.	$3x + 2y + 3y - y$	$3x + 4y$
7.	$2a - 3b + 6b + 2a$	$4a + 3b$
8.	$2x + 3y + 5x + 2y + y$	$7x + 6y$
9.	$6x - x + 6y - y$	$5x + 5y$
10.	$3a + b$	$3a + b$

B. SOLVE THE EQUATIONS FOR THE UNKNOWN AND CHECK:

1.	$7x = 35$	5
2.	$x + 5 = 15$	10
3.	$4x = 8$	2
4.	$x/2 = 9$	18
5.	$x - 3 = 6$	9
6.	$1/3\ x = 8$	24
7.	$12 = 6 + x$	6
8.	$x - 8 = 4$	12
9.	$3x = 25$	$8\frac{1}{3}$
10.	$x - 2\frac{1}{2} = 3\frac{7}{8}$	$6\frac{3}{8}$
11.	$x + 2x = 27$	9
12.	$2x + 2 = 26$	12
13.	$(2x/3) = 6$	9

14. $(x/4) + 9 = 15$	24
15. $3x + 2x - 7 = 8$	3
16. $2x - x = -8 + 10$	2
17. $(7x/3) + 6 = 8$	6/7
18. $2x - 7 + 2x = 8 + 2$	4¼
19. $6x + 4x = +6$	3/5
20. $x - \frac{3x}{4} - 2 = 8$	40
21. $x + .4 = 1.2$	.8
22. $\frac{8x}{5} - 4 = 12$	10
23. $1 - 27 = 11x - 2x - 12x$	8 2/3
24. $3x - x + 5x = 84$	12
25. $5x + 7x - 15 = +57$	6

C. WRITE THE EQUATION FOR THE FOLLOWING AND SOLVE:

1. Four times a number (x) is twenty.	5
2. Jim is six times as old as Frank. If Jim is 36 years old, how old is Frank?	6
3. John has fourteen more dollars than Ed. If Ed has 24 dollars, how much does John have?	$38.
4. Joy had twice as many problems correct as Bob. If Joy had ten problems correct how many did Bob have correct?	5
5. What number decreased by seven is equal to sixteen?	23
6. Frank spent one-third of his allowance. If Frank spent $2.50, how much did he receive for his allowance?	$7.50

UNIT 4

Supplementary Drill

Frames 1514-1970

A. MATCH THE SECOND LIST WITH THE FIRST LIST:

		ANSWER
1. πr^2	a) area of a trapezoid	i
2. $2\pi r$	b) volume of a rectangular solid	f
3. 4s	c) perimeter of a rectangle	g
4. $\frac{1}{2}h(b + b')$	d) radius of a circle	a
5. lwh	e) area of a rectangle	b
6. s^2	f) circumference of a circle	l
7. 2w + 2l	g) perimeter of a square	c
8. $\frac{1}{2}bh$	h) area of a triangle	h
9. bh	i) area of a circle	e
10. $4/3\,\pi r^3$	j) perimeter of a triangle	k
11. $\frac{1}{2}d$	k) volume of a sphere	d
12. a + b + c	l) area of a square	j

B. SOLVE THE FOLLOWING:

1. Find the length of a rectangle if its area is 72 sq.in. and the width is 8 in. — 9 in.

2. The perimeter of an isosceles triangle is equal to 32 in. If one of the equal sides is 10 inches long, how long is the unequal side? — 12 in.

3. A right triangle with legs of 8 and 11 inches has an area of how many square inches? — 44 sq. in.

4. Find the area of a circle whose diameter is 6 inches. — 28.26

5. A trapezoid with bases of 6 and 14 inches has a height of 5 inches. What is its area? — 50 sq. in.

6. What is the circumference of a circle whose radius is 3 3/4 inches? — 23.55 in.

7. A walkway $2\frac{1}{2}$ ft. wide is constructed about a rectangular garden. The garden is 20 ft. by 5 ft. How many square feet of walkway are there? — 150 sq. ft.

8. Find the height of a rectangular solid whose base is 5 inches by 14 inches, and whose volume is 1120 cubic inches. — 16 in.

9. How long will it take $240 to earn $80 at 5% interest? 6 2/3 yr.

10. Find the volume of a sphere whose diameter is 8 inches. 267.9 cu. in.

11. The resistance of an electric circuit is 120 ohms. When a voltage of 20 volts is applied to the circuit how much current flows through the circuit? .20 amp

12. What is the amount due on an $800 note at 6.5% for three-fourths of a year? $39.

13. If a car travels for four and one-half hours and during that time it travels 207 miles, what is it's average rate for this trip? 46 miles / hr.

14. An object was dropped from an airplane passing over head. If it took 12 seconds for the object to hit the ground, how high was the plane? 2304 ft.

15. If an object fell for three seconds, at what height was it dropped? 144 ft.

C. WRITE THE FORMULA FOR y IN TERMS OF x:

x	0	1	2	3	4	5
y	2	3	4	5	6	7

$y = x + 2$

x	0	1	2	3	4	5
y	1	3	5	7	9	11

$y = 2x + 1$

x	0	1	2	3	4	5
y	1	2	5	10	17	26

$y = x^2 + 1$

x	0	1	2	3	4	5
y	2	5	8	11	14	17

$y = 3x + 2$

UNIT 5

Supplementary Drill

Frames 1971-2760

A) ADD	ANSWERS	B) SUBTRACT	ANSWERS
1. $\begin{array}{r} +8 \\ \underline{+2} \end{array}$	10	6. $\begin{array}{r} +8 \\ \underline{-2} \end{array}$	10
2. $\begin{array}{r} -6 \\ \underline{+4} \end{array}$	-2	7. $\begin{array}{r} 7 \\ \underline{-2} \end{array}$	9
3. $\begin{array}{r} -8 \\ \underline{-4} \end{array}$	-12	8. $\begin{array}{r} 8 \\ \underline{6} \end{array}$	2
4. $\begin{array}{r} +6 \\ \underline{-2} \end{array}$	+4	9. $\begin{array}{r} -7 \\ \underline{-8} \end{array}$	1
5. $\begin{array}{r} -7 \\ \underline{-12} \end{array}$	-19	10. $\begin{array}{r} -10 \\ \underline{+3} \end{array}$	-13

B) COMBINE TERMS:	ANSWERS
11. +4 -2 +8 -5	+5
12. -8 -6 -5 +7	-12
13. (+8) -(+8)	0
14. +6 +2 -7 -7	-6
15. (+16) -(+2)	14

C) MULTIPLY	ANSWERS	D) DIVIDE	ANSWERS
1. 3(-2)	-6	11. $\frac{-56}{8}$	-7
2. (-6)(-2)	+12	12. $\frac{-27}{-3}$	+9
3. (-4)(+3)	-12	13. $\frac{-96}{+16}$	-6
4. (-10)(+10)	-100	14. $\frac{+28}{-7}$	-4
5. (-8)(-4)	+32	15. $\frac{-45}{-5}$	+9
6. (+2)(+7)	+14		
7. (+9)(-6)	-54		
8. (+4)(+7)	+28		
9. (-9)(+3)	-27		
10. (+7)(-6)	-42		

E. ADD:	ANSWERS
1. $\begin{array}{l} 6a + 4b + c \\ 3a - 2b - c \\ \underline{-a + \ b + 3c} \end{array}$	$8a + 3b + 3c$
2. $\begin{array}{l} 4x^3 - x^2 + x - 5xy \\ 2x^3 \qquad + 3x + 4xy \\ \underline{-x^3 - x^2 \qquad - 6xy} \end{array}$	$5x^3 - 2x^2 + 4x - 7xy$

F. SUBTRACTION:	
1. Subtract $(6x^2 - 4)$ from $(8x^2 - x + 6)$	$2x^2 - x + 10$
2. Subtract $(a^2b + b^2a - ab)$ from $(3a^2b^2 - 2a^2b + 3ab)$	$3a^2b^2 - 3a^2b + 4ab - ab^2$

G. MULTIPLY:	
1. $-5x(x^2)$	$-5x^3$
2. $(-3x)(4x^2)$	$-12x^3$
3. $(6x)(-2)(-4x^2)$	$+48x^3$
4. $7x^2y(-3xy^3)$	$-21x^3y^4$
5. $(4x)^2$	$16x^2$
6. $(3x)^2(2y)^2$	$36x^2y^2$
7. $(5x^2)^2$	$25x^4$
8. $(-4x)(3x^3)^2$	$-36x^7$
9. $(-6x)(4x)(x)^3$	$-24x^5$
10. $(-2xy)(3x^2y)(-2x)^2$	$-24x^5y^2$

H. DIVIDE:	ANSWERS		ANSWERS		ANSWERS
1. $\frac{-6x}{-2}$	$3x$	4. $\frac{x^2y^2}{-xy}$	$-xy$	8. $\frac{45x^2y}{-y}$	$-45x$
2. $\frac{6x^3}{-3x}$	$-2x^2$	5. $\frac{-36a^2b^3}{-17ab^2}$	$\frac{36ab}{17}$	9. $\frac{-24x^2y^3z^5}{-4xz}$	$6xy^3z^4$
3. $\frac{-81x^3y}{9xy}$	$-9x^2$	6. $\frac{72x^2y^5}{-8x}$	$-9xy^5$	10. $\frac{-8xy^3}{16z}$	$\frac{-xy^3}{2z}$
		7. $\frac{(3x)^2}{x}$	$9x$		

UNIT 6

Supplementary Drill   Frames 2761-3030

A. CHANGE EACH FRACTION TO ITS LOWEST EQUIVALENT TERM:

		Answer			Answer
1.	$\frac{10x}{15}$	$\frac{2x}{3}$	6.	$\frac{64a^3b^2}{-8ab^2}$	$\frac{-8a^2}{1}$
2.	$\frac{a^3b^2}{3ab}$	$\frac{a^2b}{3}$	7.	$\frac{-3x^3}{-9xy}$	$\frac{x^2}{3y}$
3.	$\frac{x^3y^2}{4y}$	$\frac{x^3y}{4}$	8.	$\frac{27x^3y}{-9xy^2}$	$\frac{-3x^2}{y}$
4.	$\frac{6x^2y^3}{8x}$	$\frac{3xy^3}{4}$	9.	$\frac{-32x^4y^5z^4}{-16x^2y^6z^4}$	$\frac{2x^2}{y}$
5.	$\frac{-12x^3y^4}{6x^2y^5}$	$\frac{-2x}{y}$	10.	$\frac{48x^3y^2}{-7x^3z^2}$	$\frac{48y^2}{-7z^2}$

B. MULTIPLY:

		Answer
1.	$4(x + y)$	$4x + 4y$
2.	$x(x + 2y + w)$	$x^2 + 2xy + xw$
3.	$-3a(a + b + 2c)$	$-3a^2 - 3ab - 6ac$
4.	$-xy(x^2 + x - 3)$	$-x^3y - x^2y + 3xy$
5.	$-4ab(3a + 2b^2)$	$-12a^2b - 8ab^3$
6.	$x^2y(x^3 + xy + y)$	$x^5y + x^3y^2 + x^2y^2$
7.	$4xy^3(x + y^2 + z^3)$	$4x^2y^3 + 4xy^5 + 4xy^3z^3$
8.	$-3x^2y(x^3y - 2x - 4)$	$-3x^5y^2 + 6x^3y + 12x^2y$
9.	$-a^3b^2(-a^2 - b^2 - c^2)$	$+a^5b^2 + a^3b^4 + a^3b^2c^2$
10.	$-6a^3b^2(a^3b + ab^3 - ab)$	$-6a^6b^3 - 6a^4b^5 + 6a^4b^3$

C. REMOVE PARENTHESES AND COMBINE TERMS:

1.	$(11x - 5)$	$11x - 5$
2.	$-(3x + 4)$	$-3x - 4$
3.	$5x - (2x + 5)$	$3x - 5$
4.	$3a - (2a - 4)$	$a + 4$

5. $4 +(y - 5)$	$y - 1$
6. $7y - 4 - 2(y - 2)$	$5y$
7. $8b - 3(b + 4)$	$5b - 12$
8. $(x - y) + (3x + y)$	$4x$
9. $7a +(-2a + 3)$	$5a + 3$
10. $2x +(x - 6) + 9$	$3x + 3$
11. $(3x + 2y) - (5x + 6y)$	$-2x - 4y$
12. $3a -(6a - 4)$	$4 - 3a$
13. $a -(-3 - 2a)$	$3a + 3$
14. $3x - 2(3x + 4)$	$-3x - 8$
15. $a(a - b) + b(a + b)$	$a^2 + b^2$
16. $3 - [-4a + 2(a + 1)]$	$2a + 1$
17. $2a - 3[a + 2(3a - 2)]$	$12 - 19a$
18. $4x + [x + 3(x + 2) -3]$	$8x + 3$
19. $-[3x -(x - y)]$	$-2x - y$
20. $2a + 6(a + 1) -[a -(a + 1)]$	$8a + 7$
21. $3x + x[x + 2(x + 6)]$	$3x^2 + 15x$
22. $m^2 - nm + n[m - n(m - n)]$	$m^2 - n^2m + n^3$
23. $2 -[3x - 2(x - 2) - 4]$	$2 - x$
24. $-3[x -(2 - x)] - [x -(x + 4)]$	$-x^2 - 5x + 10$
25. $4[a - 2(b - a)] +[a - b(a - 2)]$	$-13a - 6b - ab$

UNIT 7

Supplementary Drill Frames 3031-3356

A. DIVIDE:	Answer
1. $\frac{xyz + xy}{x}$	$yz + y$
2. $\frac{abc - 3ab}{ab}$	$c - 3$
3. $\frac{2a^3 - a^2 - ab}{a}$	$2a^2 - a - b$
4. $\frac{4x^4 - 2x^3 + 16x}{2x}$	$2x^3 - x^2 + 8$
5. $\frac{8x^5 - 10x^4 + x^3}{x^3}$	$8x^2 - 10x + 1$
6. $\frac{2b^8 + 4b^9 + 2b^{10}}{2b^7}$	$b + 2b^2 + b^3$
7. $\frac{9a^{10} - 9a^9}{9a}$	$a^9 - a^8$
8. $\frac{-5x - 30x^{10}}{5x}$	$-1 - 6x^9$
9. $\frac{18a^3 + 12a^2 - 9a}{-3a}$	$-6a^2 - 4a + 3$
10. $\frac{6x^7 - 4x^5}{-2x^4}$	$-3x^3 + 2x$
11. $\frac{a^2x^3y - 10ax^5y^2}{ax^2y}$	$ax - 10x^3y$
12. $\frac{a - 2a^2 + 3a^3}{a}$	$1 - 2a + 3a^2$
13. $\frac{ab + ab^2 + ab}{ab}$	$b + 2$
14. $\frac{4x^3y^2z - 14x^2yz^5}{2xyz}$	$2x^2y - 7xz^4$
15. $\frac{a^3b - 4ab^3 + 5a^2b^2}{ab}$	$a^2 - 4b^2 + 5ab$

B. MULTIPLE	
1. $\begin{array}{l} a + 5 \\ \underline{a - 5} \end{array}$	$a^2 - 25$

2.	$8 - vw$ $\underline{8 - vw}$	$64 - 16vw + v^2w^2$
3.	$g - 2$ $\underline{g + 7}$	$g^2 + 5g - 14$
4.	$x + 7$ $\underline{x - 10}$	$x^2 - 3x - 70$
5.	$4y - 3$ $\underline{2y + 2}$	$8y^2 + 2y - 6$
6.	$x - 10$ $\underline{x - 1}$	$x^2 - 11x + 10$
7.	$a - 7$ $\underline{a - 9}$	$a^2 - 16a + 63$
8.	$2x + 1$ $\underline{3x + 2}$	$6x^2 + 7x + 3$
9.	$3x - 4$ $\underline{5x + 2}$	$15x^2 - 14x - 8$
10.	$4y - 1$ $\underline{y + 2}$	$4y^2 + 7y - 2$

C.	DIVIDE:	ANSWER
1.	$(x^3 + 8x^2y + 14xy^2 - 8y^3) \div (x + 4y)$	$x^2 + 4xy - 2y^2$
2.	$(x^2 - 3x - 28) \div (x - 7)$	$x + 4$
3.	$(y^2 - y - 20) \div (y + 4)$	$y - 5$
4.	$(20a^2 - 13a + 2) \div (4a - 1)$	$5a - 2$
5.	$(-3x^2 + 13x - 15) \div (-3x + 7)$	$x - 2$, R: -1
6.	$(x^2 - 9) \div (x + 3)$	$x - 3$
7.	$(a^3 - 1) \div (a - 1)$	$a^2 + a + 1$
8.	$(6y^2 + 34y + 24) \div (2y + 10)$	$3y + 2$, R: 4
9.	$(3n^2 - 17mn + 10m^2) \div (3n - 2m)$	$n - 5m$
10.	$(12x^3 + 4x^2 - 43x + 120) \div (2x - 5)$	$6x^2 + 17x + 21$, R: 225

D. SIMPLIFY AND WRITE THE ANSWERS USING NEGATIVE EXPONENTS:

	Answers		Answers
1. $\frac{x}{x^2}$	x^{-1}	4. $\frac{mn}{m^5n^3}$	$m^{-4}n^{-2}$
2. $\frac{y^3}{y^5}$	y^{-2}	5. $\frac{-5xy}{x^3y^2}$	$-5x^{-2}y^{-1}$
3. $\frac{1}{a}$	a^{-1}		

UNIT 8

Supplementary Drill

Frames 3358-3654

A. SOLVE THE FOLLOWING EQUATIONS:

1.	$5x = 3x + 8$	4
2.	$2x - 8 = 7 - 3x$	3
3.	$7x - 2x = x - 8$	-2
4.	$8x - 12 + 2x = 4x + 30$	7
5.	$14 -(8 - 2y) = 5$	$-\frac{1}{2}$
6.	$2x + 2(x - 2) = 8$	3
7.	$8x -(x - 4) = 6(x + 6)$	32
8.	$3y -(2y - 2) = 2(6 - y)$	10/3
9.	$4x +(x - 8) = 10 -(3x + 7)$	11/8
10.	$5x - 2(x + 6) = 7 - (2x + 1)$	4

A student that is having difficulties in solving equations should be referred back to frames 976-1513 and sec. B of the Unit 3 supplementary problems.

B. SOLVE THE FOLLOWING EQUATION FOR THE SPECIFIED LETTER.

1.	$I = prt$ for t	$t = I/pr$
2.	$D = rt$ for r	$r = D/t$
3.	$C = 2\ r$ for r	$r = C/2$
4.	$V = lwh$ for w	$w = V/lh$
5.	$A = \frac{1}{2}h(b + b\ )$ for h	$h = 2A/(b + b\ )$
6.	$x + 2b = 5b$ for x	$x = 3b$
7.	$3a - 2b + a = 2b - a$ for a	$a = 4b/5$
8.	$a - 3b(b + 2) = 3a - b$ for a	$a = \frac{-3b^2 - 5b}{2}$
9.	$2x - 3(x - y) = x - 2y$ for x	$x = 5y/2$
10.	$3a - 3(a - b) + a = 2a - b$ for a	$a = 4b$

11. $3 + 6k + 4 = 1 - 2k + 3 - k$, for k $k = -1/3$

12. $6 - 4(3f - 5) = 37 +(f - 7)$, for f $f = -4/13$

13. $5(x + 3) - 3(2 + x) + 5(x + 7) = 5(x + 3)$, for x $x = -29/2$

14. $3(m + 2) - 5m = 9 - m + 2(m + 1)$, for m $m = -5/3$

15. $3 - 4(t - \frac{1}{2}) = 2/5(t + 7) + 5$, for t $t = -7/11$

UNIT 9

Supplementary Drill **Frames 3655-4027**

A. MAKE A TABLE OF VALUES AND PLOT THE FOLLOWING EQUATIONS ON GRAPH PAPER:

1. $x + y = 4$
2. $2x - y = 7$
3. $3x + y = -4$
4. $y - 2x = 6$
5. $2y - x = -4$

B. WITHIN WHICH QUADRANT DOES EACH OF THE FOLLOWING POINTS LIE?

		ANSWER
1.	(2, 3)	I
2.	(-3, -2)	III
3.	(-2, 6)	II
4.	(3, -2)	IV
5.	(0, 0)	none

C. FOR EACH OF THE FOLLOWING EQUATIONS DETERMINE THE SLOPE AND THE y-INTERCEPT OF THE CORRESPONDING STRAIGHT LINE:

		ANSWER
1.	$y = 2x + 4$	$m = 2, 4$
2.	$y = -\frac{3}{4}x + 7$	$m = -3/4, 7$
3.	$2x - \frac{1}{2}y = 8$	$m = 4, -16$
4.	$10 = -3y + 2x$	$m = 2/3, -\frac{10}{3}$
5.	$\frac{1}{2}x = 1/3y - 4$	$m = 3/2, 12$

D. CONSTRUCT A BROKEN LINE GRAPH USING THE FOLLOWING INFORMATION:

Days	1	2	3	4	5	6	7	8	9	10
Temp.	68°	55°	63°	72°	64°	67°	64°	58°	60°	58°

E. GRAPH THE FOLLOWING SETS OF EQUATIONS AND FIND THEIR POINT OF INTERSECTION:

		ANSWER
1.	$x + y = 2$ $x - y = 4$	3, -1
2.	$4x + y = 10$ $x + 2y = -8$	4, -6

3.	$x = 4$ $3y - 2x = 1$	4, 3
4.	$y = -3$ $3x = y + 6$	1, -3
5.	$7x - 6y = 24$ $9x - 8y = -32$	0, -4
6.	$2x - y = 2$ $x + 3y = 22$	4, 6
7.	$3x + 4y = 6$ $x - y = 2$	2, 0
8.	$2x - 3y = 5$ $5x - 4y = 9$	1, -1
9.	$x - 2y = 5$ $4x + 2y = 6$	2, -1
10.	$2x + 3y = 8$ $-x + 2y = 6$	-2/7, 2 6/7

UNIT 10

Supplementary Drill

Frames 4028-4300

A. SOLVE THE FOLLOWING SETS OF EQUATIONS FOR THE TWO UNKNOWN.

1. $x + y = 6$ $x - y = -1$	$x = 2\frac{1}{2}$ $y = 3\frac{1}{2}$
2. $y = 3x$ $y = -3x + 4$	$x = 2/3$ $y = 2$
3. $3x + y = 9$ $2x - y = 1$	$x = 2$ $y = 3$
4. $c = d - 4$ $3c + d = 4$	$c = 0$ $d = 4$
5. $3x - y = -7$ $2x - y = 5$	$x = -12$ $y = -29$
6. $2m - 3n = -9$ $5m - 2n = 5$	$m = 3$ $n = 5$
7. $2x + 3y = 6$ $2x - 5y = 22$	$x = 6$ $y = -2$
8. $3a + 4b = 22$ $a + 2b = 8$	$a = 6$ $b = 1$
9. $4a - 3b = 8$ $3a + 2b = 18$	$a = 4\ 2/17$ $b = 2\ 14/17$
10. $.5x - y = 1.4$ $.3x + 2y = 2.4$	$x = 4$ $y = .6$

UNIT 11

Supplementary Drill — Frames 4301-4601

A.	SOLVE:	ANSWERS
1.	Find two consecutive odd integers whose sum is 76.	37, 39
2.	Find two consecutive even integers whose sum is 94.	46, 48
3.	The sum of three consecutive integers is 102. Find the integers.	33, 34, 35
4.	The sum of two numbers is 20. Twice the first is three times the other. Find the numbers.	8, 12
5.	The greater of two numbers is 9 times the smaller. The difference of the two numbers is 48. What are the two numbers?	6, 54
6.	Find two numbers such that 3 times the smaller is 7 less than twice the larger, and if twice the larger is 8 more than the sum of the two numbers.	9, 17
7.	Find the numbers such that one is as much more than 30 as the other is less than 30, and eight times their difference is equal to the larger.	28, 32
8.	Two cars start 1200 miles apart and travel towards each other. One travels at the rate of 44 miles per hour and the other at 56 miles per hour. How far will each car travel before they meet?	672, 528
9.	A plane and car both leave the same city traveling in the same direction. The car leaves first and travels at the rate of 45 miles per hour, and 4 hours later the plane leaves traveling at the rate of 75 miles per hour. How far will each travel before the plane overtakes the car?	450 mi.
10.	Two planes leave an airport and travel in opposite directions. One plane is traveling at a rate of 10 miles per hour faster than the other. What is the rate of each plane if after 6 hours they are 960 miles apart?	75 m.p.h. 85 m.p.h.

UNIT 12

Supplementary Drill

Frames 4602-5081

A. SIMPLIFY:	ANS.
1. $\sqrt{121}$	11
2. $\sqrt{x^{10}}$	x^5
3. $\sqrt{x^4y^8}$	x^2y^4
4. $\sqrt{a^4b^6c^2}$	a^2b^3c
5. $\sqrt{64a^2}$	$8a$
6. $\sqrt{225x^8}$	$15x^4$
7. $\sqrt{36x^4y^2}$	$6x^2y$
8. $\sqrt{16x^6y^{10}}$	$4x^3y^5$
9. $\sqrt{256y^{18}}$	$16y^9$
10. $\sqrt{289x^4y^6z^{14}}$	$17x^2y^3z^7$
B. MULTIPLY:	
1. $(a + 6)(a - 6)$	$a^2 - 36$
2. $(2x + 3)(2x - 3)$	$4x^2 - 9$
3. $(3x + 7)(3x - 7)$	$9x^2 - 49$
4. $(5x - 6)(5x + 6)$	$25x^4 - 36$
5. $(4a - 3)(2a + 3)$	$8a^2 + 6a - 9$
6. $(3y - 5)(y + 2)$	$3y^2 + y - 10$
7. $(4y - 5)(2y - 3)$	$8y^2 - 22y + 15$
8. $(m - 7)(m + 4)$	$m^2 - 3m - 28$
9. $(5x + 2)(3x + 4)$	$15x^2 + 26x + 8$
10. $(8 - x)(5 + 2x)$	$40 + 11x - 2x^2$
C. FACTOR:	
1. $16 - 8x$	$8(2 - x)$

2. $x^2 - 9$	$(x - 3)(x + 3)$
3. $121x^2 - 16$	$(11x - 4)(11x + 4)$
4. $64x^2 - y^2$	$(8x - y)(8x + y)$
5. $36a^2 - 49b^2$	$(6a - 7b)(6a + 7b)$
6. $25x^2 - 169y^4$	$(5x - 13y\ )(5x + 13y\ )$
7. $a^3 - 25a$	$a(a + 5)(a - 5)$
8. $8x^3 - 72xy$	$8x(x - 3y)(x + 3y)$
9. $16a^4 - 81b^4$	$(4a\ \ + 9b^2)(2a - 3b)(2a + 3b)$
10. $4x^2 - 36y^2$	$4(x - 3y)(x + 3y)$

UNIT 13

Supplementary Drill Frames 5082-5283

A. FACTOR:	ANSWERS
1. $a^2 - 5a - 24$	$(a - 8)(a + 3)$
2. $x^2 - 15x + 36$	$(x - 13)(x - 2)$
3. $a^2 - 7a - 30$	$(a - 10)(a + 3)$
4. $x^2 + 5x - 24$	$(x + 8)(x - 3)$
5. $n^2 - 11n + 30$	$(n - 5)(n - 6)$
6. $x^2 - 3xy - 40y^2$	$(x - 8y)(x + 5y)$
7. $2a^2 + 7ab - 15b^2$	$(2a - 3b)(a + 5b)$
8. $5b^2 - 15b - 50$	$5(b + 5)(b - 2)$
9. $3x^2 + 6x - 9$	$3(x - 1)(x + 3)$
10. $4x^2 - 20x + 25$	$(2x - 5)(2x - 5)$
11. $9 + 24x + 16x^2$	$(3 + 4x)(3 + 4x)$
12. $a^2 - 10ab + 25b^2$	$(a - 5b)(a - 5b)$
13. $40n^2 + n - 6$	$(8n - 3)(5n + 2)$
14. $12x^2 - 13x - 14$	$(3x + 2)(4x - 7)$
15. $6x^2 - 35xy - 6y^2$	$(6x + y)(x - 6y)$
16. $15a^2 - 8a + 1$	$(3a - 1)(5a - 1)$
17. $4x^2 - 20xy + 25y^2$	$(2x - 5y)(2x - 5y)$
18. $3a^2 - 10ab + 3b^2$	$(3a - b)(a - 3b)$
19. $2x^3 - 16x^2 - 18x$	$2x(x - 9)(x + 1)$
20. $4.5a^2 + 2.4a + .3$	$.3(3a + 1)(5a + 1)$
21. $72 - m - m^2$	$(9 + m)(8 - m)$
22. $2x^2 + 7x + 3$	$(2x + 1)(x + 3)$
23. $8y^2 - 15y - 2$	$(y - 2)(8y + 1)$
24. $12x^2 + 7x - 12$	$(3x + 4)(4x - 3)$

25. $6a^2 - 40a - 14$	$(6a + 2)(a - 7)$
26. $3b^2 + 22b - 16$	$(3b - 2)(b + 8)$
27. $6s^2 - 15s - 9$	$3(s - 3)(2s + 1)$
28. $21x^2 + 22x - 8$	$(3x + 4)(7x - 2)$
29. $18s^2t - 60st + 50t$	$2t(3s - 5)(3s - 5)$
30. $3y^6 - 78y^4 + 75y^2$	$3y^2(y + 5)(y - 5)(y + 1)(y - 1)$

UNIT 14

Supplementary Drill Frames 5285-5461

A. SOLVE FOR x:	ANSWERS
1. $x^2 - x - 12 = 0$	4, -3
2. $x^2 + 10x + 24 = 0$	-6, -4
3. $3x^2 - 6x = 0$	0, 2
4. $x^2 + 9x - 36 = 0$	-12, 3
5. $3x^2 + 4x + 1 = 0$	-1, -1/3
6. $8x^2 + 2x - 1 = 0$	$-\frac{1}{2}, \frac{1}{4}$
7. $2x^2 - 10x + 12. = 0$	2, 3
8. $5x^2 + 15x - 50 = 0$	2, -5
9. $12x^2 - 26x + 12 = 0$	2/3, 3/2
10. $49x^2 + 70x + 25 = 0$	-5/7, -5/7

B. SOLVE FOR x:	ANSWERS
1. Find three consecutive even integers whose sum is 42.	12, 14, 16
2. Find four consecutive integers whose sum is 162.	39, 40, 41, 42
3. The product of two consecutive positive odd integers is 575. Find the numbers.	23, 25
4. Find the number whose square is 18 more than 7 times the number.	9, -2
5. One number is 4 greater than another, their product is 357. Find the numbers.	17, 21
6. The sum of two numbers is 30. If the product of these numbers is equal to 216, what are the numbers?	18, 12
7. Three times a number is 10 less than the square of the number. What is the number?	5 or -2
8. A number increased by four; multiplied by the number decreased by -6 is equal to the square of the number diminished by 48.	12

UNIT 15

Supplementary Drill

Frames 5462-6150

A. REDUCE TO LOWEST TERMS:	ANSWER
1. $\frac{15r^2s}{12rs}$	$\frac{5r}{4}$
2. $\frac{5\pi r^3}{25\pi r^2}$	$\frac{r}{5}$
3. $\frac{x^2 - x}{x^2 + x}$	$\frac{x - 1}{x + 1}$
4. $\frac{a^2 + ab}{9ay}$	$\frac{a + b}{9y}$
5. $\frac{(p - 9)^2}{p^2 - 9^2}$	$\frac{p - 9}{p + 9}$
6. $\frac{2ab + 2b}{a^2 - 1}$	$\frac{2b}{a - 1}$
7. $\frac{7a^2b - 28b}{a^2 - 8a + 12}$	$\frac{7b(a + 2)}{a - 6}$
8. $\frac{x^2 - 3x - 4}{2x^2 - 4x - 16}$	$\frac{x + 1}{2(x + 2)}$
9. $\frac{a^2 - 4b^2}{a^2 - 4ab + 4b^2}$	$\frac{a + 2b}{a - 2b}$
10. $\frac{4x^2 + 20x + 25}{2x^2 + 9x + 10}$	$\frac{2x + 5}{x + 2}$

B. MULTIPLY AND REDUCE ANSWER TO LOWEST TERMS:	ANSWER
1. $\frac{3a}{5b} \cdot \frac{10}{6a^2}$	$\frac{1}{ab}$
2. $\frac{xy}{5} \cdot \frac{1}{x^2}$	$\frac{y}{5x}$
3. $\frac{27ab}{2xy} \cdot \frac{9x^2y^2}{ab^2}$	$\frac{243xy}{2b}$
4. $\frac{3x^2}{2a} \cdot \frac{6ay}{xy}$	$\frac{9x}{y}$
5. $\frac{m^2 - 9}{3m} \cdot \frac{6m}{m + 3}$	$2(m - 3)$

6. $\frac{a^2 - b^2}{a^2} \cdot \frac{a^4}{(a + b)^2}$	$\frac{a^2 (a - b)}{a + b}$
7. $\frac{x^2 + 9x + 18}{x^2 + 6x + 9} \cdot \frac{xy - 4y}{x^2 - 9x + 20}$	$\frac{y(x + 6)}{(x + 3)(x - 5)}$
8. $\frac{m^2 - 2mn - 15n^2}{m^2 - 7mn + 10n^2} \cdot \frac{m^2 - mn - 2n^2}{m^2 + 4mn + 3n^2}$	1
9. $m^2 - n^2 \cdot \frac{m}{m^2 + 2mn + n^2}$	$\frac{m(m - n)}{m + n}$
10. $\frac{s^2 + 5 - 2}{s^2 - 1} \cdot \frac{5s^2 - 12s + 4}{s^2 - 4}$	$\frac{5s - 2}{s + 1}$

C. DIVIDE AND REDUCE ANSWERS TO LOWEST TERMS:	ANSWER
1. $\frac{5}{6x} \div \frac{3}{4x}$	$\frac{10}{9}$
2. $\frac{a - 3}{2a + 5} \div \frac{a + 2}{2a + 5}$	$\frac{a - 3}{a + 2}$
3. $\frac{4a^2b}{3a^2b} \div \frac{8a^2b^2}{7abc}$	$\frac{7c}{6a}$
4. $\frac{x^2 - 1}{x^2 + 7x + 6} \div \frac{x^2 - 2x + 1}{x + 6}$	$\frac{1}{x - 1}$
5. $\frac{3b - 6}{5b^2} \div \frac{b^2 - 4}{b + 2}$	$\frac{3}{5b}$
6. $\frac{c^2 - 25}{5c + 10} - \frac{c^2 - 8 + 15}{c^2 - c - 6}$	$\frac{c + 5}{5}$
7. $\frac{9x^2 - 4y^2}{x + y} \div \frac{3x + 2y}{x^2 - y^2}$	$(3x - 2y)$
8. $\frac{a - b}{a^2 + 2ab + b^2} \div \frac{2a - 2b}{4}$	$\frac{2}{(a + b)(a + b)}$
9. $\frac{ab^2 + a^2b}{cb^2 + c^3} \div \frac{3b + 3a}{b^4 - c^4}$	$\frac{ab(b - c)(b + c)}{3c}$
10. $\frac{2m^2 + 15m + 25}{3m^2 + 10m - 25} \div \frac{m^2 + 7m + 10}{3m^2 - 14m + 15}$	$\frac{(2m + 5)(m - 3)}{(m + 5)(m - 2)}$

D. COMBINE AND REDUCE TO LOWEST TERMS:	ANSWER
1. $\frac{a}{2} + \frac{a}{3} + \frac{a}{4}$	$\frac{13a}{12}$
2. $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$	$\frac{bc + ac + ab}{abc}$

3.	$\frac{5}{a} - \frac{4}{a^2} - \frac{2}{a^3}$	$\frac{5a^2 - 4a - 2}{a^3}$
4.	$\frac{1}{3(x + 1)} - \frac{1}{4(x + 1)}$	$\frac{1}{12(x + 1)}$
5.	$\frac{P}{9 - r} + \frac{P}{r - 9}$	0
6.	$x + 1 - \frac{x^2 - x - 18}{x - 3}$	$\frac{15 - x}{x - 3}$
7.	$\frac{3x}{x^2 + x} - \frac{2}{x + 1}$	$\frac{1}{x + 1}$
8.	$\frac{m - n}{(m + n)^2} + \frac{n}{m^2 - n^2}$	$\frac{m^2 - mn + 2n^2}{(m + n)(m + n)(m - n)}$
9.	$\frac{x}{x + y} - \frac{xy}{(x + y)^2} - \frac{xy^2}{(x + y)^3}$	$\frac{x(x^2 + xy - y^2)}{(x + y)^3}$
10.	$\frac{a - 2}{a^2 + 2a + 1} - \frac{a - 1}{a^2 - a - 2}$	$\frac{5 - 4a}{(a + 1)^2(a - 2)}$
11.	$\frac{3}{x - 1} + \frac{2}{2x + 2} + \frac{2x^2}{x^2 - 1}$	$\frac{2(x + 1)}{x - 1}$
12.	$\frac{3}{y^2 - 4y + 4} + \frac{y}{y^2 - 4} - \frac{y}{y^2 + 4y + 4}$	$\frac{7y^2 + 4y + 12}{(y - 2)^2(y + 2)^2}$
13.	$\frac{x - 3}{x^2 - 7x + 10} + \frac{x + 1}{x^2 - 4}$	$\frac{2x^2 - 5x - 11}{(x + 2)(x - 2)(x - 5)}$
14.	$\frac{w + 4}{w^2 + 3w - 10} - \frac{w - 4}{w^2 - 5w + 6}$	$\frac{8}{(w + 5)(w - 2)(w - 3)}$
15..	$\frac{c + 2}{c^2 + 2c - 3} + \frac{c - 4}{c^2 - 6c + 5} - \frac{c + 6}{c^2 - 2c - 15}$	$\frac{c^2 - 9c - 16}{(c + 3)(c - 1)(c - 5)}$

UNIT 16

Supplementary Drill

Frames 6151-6331

A. TELL WHETHER THE FOLLOWING FRACTIONS ARE INCREASING OR DECREASING UNDER EACH SPECIFIED CONDITION:

		ANSWER
1.	3/m, m increasing and not equal to zero.	decreasing
2.	2t/4, t increasing.	increasing
3.	0/a, a increasing and not equal to zero.	doesn't change
4.	b/2c, b decreasing, c constant	decreasing
5.	8/c, c decreasing and not equal to zero.	increasing
6.	$\frac{3a}{b+1}$, b increasing, b not equal to -1, a constant	decreasing
7.	$\frac{4x+2}{8}$, x increasing	increasing
8.	$\frac{3+a}{y}$, y decreasing, y not equal to zero	increasing
9.	$\frac{4b}{8}$, b decreasing	decreasing
10.	$\frac{3ab}{c}$, c increasing, c not equal to zero, a, b constant	decreasing

B. SOLVE THE FOLLOWING EQUATIONS FOR THE UNKNOWN:

		ANSWER
1.	$\frac{x}{2} = \frac{x+1}{3}$	$x = 2$
2.	$\frac{r-1}{r} = \frac{7}{5}$	$r = -5/2$
3.	$\frac{m-1}{m+1} = \frac{m}{m-1}$	$m = 1/3$
4.	$\frac{x+3}{4} = \frac{x^2+7}{4x}$	$x = 7/3$
5.	$\frac{10}{16-b} = \frac{8}{4-b}$	$b = -44$
6.	$\frac{1}{y^2+7y-3} = \frac{1}{10+11y+y^2}$	no roots
7.	$a - 3 = \frac{a^2+4}{a-1}$	$a = -\frac{1}{4}$

8. $\frac{3y}{2y + 3} - \frac{y}{3 - 2y} = \frac{8y^2 + 12}{4y^2 - 9}$ $x = -2$

9. $\frac{5x}{x + 2} = \frac{6}{x + 3} + 5$ $x = -2\ 1/8$

10. $\frac{4x + 5}{x^2 + 13x + 42} = \frac{2x}{x^2 + 4x - 21} + \frac{2}{x - 3}$ $x = -2\ 1/5$

WRITE AS A FRACTION:	ANSWER
1. a^{-1}	$1/a$
2. xy^{-1}	x/y
3. $c^2d^{-1}e^{-3}$	c^2/de^3
4. a^2b^{-4}	a^2/b^4
5. $m^{-2}n^{-3}$	$1/m^2n^3$

UNIT 17

Supplementary Drill

Frames 6332-6511

SOLVE THE FOLLOWING PROBLEMS:

1. The sum of twice a number plus one-third of the number is 84. What is the number? 36

2. The difference between one-half a number and one-third a number is 30. What is the number? 180

3. Separate 96 into two parts such that the smaller is one-third the large part. 72, 24

4. Separate 50 into two parts such that four-sevenths of the larger is equal to three times the smaller. 42, 8

5. Three times a number diminished by one-fourth of the number is equal to 33. What is the number? 12

6. The sum of two numbers is 144. When the larger is multiplied by 2, it is equal to ten times the smaller. What are the numbers? 120, 24

7. A manufacturer of cotton wishes to sell a 1000 pound mixture of cotton for \$.22 a pound. How many pounds of a \$.12 and \$.40 sample must he use?
   357.5 lbs. @ \$.40
   642.5 lbs. @ .12

8. A grocer makes a blend of 10 pounds of coffee. In making this blend he used six pounds of a blend at \$.54 a pound and another blend at \$.84 a pound. What was the cost of the new blend per pound? \$.66 per pound

9. A man has 40 gallons of a 24% nitric acid solution. How many gallons of an 80% nitric acid solution must he add in order to have a 36% solution? $10\frac{10}{11}$ gal.

10. A chemist prepares 50 quarts of a 60% boric acid solution. What amounts of a 45% solution and a 70% solution must he use?
    20 qts. of 45%
    30 qts. of 70%

11. One boy can paint a fence in 12 days and another boy can paint the same fence in 9 days. If the two boys work together, how long will it take them to paint the fence? 5 1/7 days

12. A tank has three pipes connected to it. Two of the pipes can fill the tank in 8 and 6 hours respectively. The third pipe can drain the tank in 12 hours. How long will it take to fill the tank if all three pipes are operating? 4.8 hours

13. If three boys work together it takes four hours to wash and wax a car. If the first boy works twice as fast as the second, and the second three times as fast as the third, how long would it take each boy by himself to do the job? 4/7 hr., 1 1/7 hr., 2 4/7 hr.

14. One man can do a job in 16 hours. After he has been working for 5 hours, another man joins him and together they finish the job in 3 more hours. How long would it take the second man to do the job by himself? 6 hours

15. It takes two men working together 8 hours to finish digging a ditch. If after working for 3 hours one of the men quits, how long will it take the other man to finish the job if he works three times as hard as the first man? $1\frac{1}{4}$ hours

UNIT 18

Supplementary Drill

Frames 6512-6766

A. Simplify:	Answer
1. $\sqrt{27}$	$3\sqrt{3}$
2. $\sqrt{144}$	12
3. $\sqrt{1200}$	$20\sqrt{3}$
4. $\sqrt{48}$	$4\sqrt{3}$
5. $\sqrt{a^4b^6}$	a^2b^3
6. $\sqrt{a^3b^4}$	$ab^2\sqrt{a}$
7. $\sqrt{25x^2y}$	$5x\sqrt{y}$
8. $\sqrt{50a^7b^4c^3}$	$5a^3b^2c\sqrt{2ac}$
9. $4\sqrt{28a^5}$	$8a^2\sqrt{7a}$
10. $\frac{1}{2}\sqrt{36m^2n^8}$	$3mn^4$
11. $\sqrt{1/16}$	$\frac{1}{4}$
12. $\sqrt{1/3}$	$1/3\sqrt{3}$
13. $\sqrt{\frac{1}{2}x^2}$	$\frac{x}{2}\sqrt{2}$
14. $\sqrt{(3/5)x^2y^3}$	$1/5xy\sqrt{15y}$
15. $\sqrt{\frac{7x}{3}}$	$1/3\sqrt{21x}$

B. COMBINE:	Answer
1. $2\sqrt{5} + 3\sqrt{5} - 4\sqrt{5}$	$\sqrt{5}$
2. $\sqrt{50} + \sqrt{72}$	$11\sqrt{2}$
3. $2\sqrt{48} + \sqrt{3} + \sqrt{243}$	$18\sqrt{3}$
4. $\sqrt{1/5} + \sqrt{125} - \sqrt{80}$	$6/5\sqrt{5}$
5. $2\sqrt{12a} - \sqrt{75a} + \sqrt{3a}$	0
6. $2\sqrt{\frac{1}{2}} + \sqrt{8}$	$3\sqrt{2}$
7. $\sqrt{4} + \sqrt{96} + \sqrt{600}$	$2 + 14\sqrt{6}$

8. $\sqrt{2/3} - \sqrt{3/2} - 6$	$-(7/6)\sqrt{6}$
9. $\sqrt{20} - \sqrt{162} + \sqrt{98}$	$2\sqrt{5} - 2\sqrt{2}$
10. $\sqrt{80} + \sqrt{54} - \sqrt{24}$	$4\sqrt{5} - \sqrt{6}$

C. MULTIPLY:	ANSWER
1. $2\sqrt{3} \cdot \sqrt{6}$	$6\sqrt{2}$
2. $\sqrt{12} \cdot \sqrt{5}$	$2\sqrt{15}$
3. $3\sqrt{6} \cdot 4\sqrt{2}$	$24\sqrt{3}$
4. $\sqrt{2}\ (2\sqrt{6} + 4)$	$4\sqrt{3} + 4\sqrt{2}$
5. $(\sqrt{5} + 2)(\sqrt{5} - 2)$	1
6. $(\sqrt{3} + 4)(\sqrt{3} - 6)$	$-2\sqrt{3} - 21$
7. $(2\sqrt{6} - 3)(\sqrt{2} + 2)$	$4\sqrt{3} - 3\sqrt{2} + 4\sqrt{6} - 6$
8. $(4 - \sqrt{6})(8 + \sqrt{6})$	$26 - 4\sqrt{6}$
9. $(\sqrt{72} - 3)(\sqrt{2} + 8)$	$45\sqrt{2} - 12$
10. $(3\sqrt{15} - 2)(\sqrt{15} + 3)$	$39 + 7\sqrt{15}$

D. DIVIDE:	ANSWER
1. $\dfrac{\sqrt{18}}{\sqrt{6}}$	$\sqrt{3}$
2. $\dfrac{8\sqrt{12}}{4\sqrt{3}}$	4
3. $\dfrac{12\sqrt{xy}}{3\sqrt{x^3}}$	$\dfrac{4\sqrt{y}}{x}$
4. $\dfrac{\sqrt{32x^3}}{\sqrt{4x}}$	$2x\sqrt{2}$
5. $\dfrac{\sqrt{48} - \sqrt{6}}{2\sqrt{6}}$	$\dfrac{2\sqrt{2} - 1}{2}$

E. SOLVE FOR x:	ANSWER
1. $\sqrt{x} - 1 = 5$	36
2. $2\sqrt{x} + 2 = 6$	4
3. $\sqrt{x - 3} = 4$	19
4. $\sqrt{2x + 2} = 6$	17

5. $6 + 2\sqrt{x+3} = 8$	-2

F. WRITE IN EXPONENTIAL FORM:	ANSWER
1. $\sqrt{xy^3}$	$x^{1/2}y^{3/2}$
2. $\sqrt{a^2b^3}$	$ab^{3/2}$
3. $\sqrt{m^4n^5}$	$m^2n^{5/2}$
4. $\sqrt{2a^3b^5}$	$2^{1/2}\ a^{3/2}\ b^{5/2}$
5. $\sqrt{49a^5b^7}$	$7a^{5/2}b^{7/2}$

G. WRITE IN RADICAL FORM:	ANSWER
1. $x^{\frac{1}{2}}$	$\sqrt{x}$
2. $x^{3/2}y^{5/2}$	$\sqrt{x^3y^5}$
3. $a^{13/2}\ b^{7/2}\ c^{3/2}$	$\sqrt{a^{13}b^7c^3}$
4. $(2a)^{1/2}$	$\sqrt{2a}$
5. $(mn)^{3/2}$	$\sqrt{m^3n^3}$

UNIT 19

Supplementary Drill Frames 6767-6931

A. SOLVE THE FOLLOWING BY COMPLETING THE SQUARE:	ANSWER
1. $x^2 + 5x - 6 = 0$	$x = -6$, $x = 1$
2. $x^2 - 2x - 7 = 0$	$x = 1 + 2\sqrt{2}$, $x = 1 - 2\sqrt{2}$
3. $5t^2 + 7t - 6 = 0$	$t = -2$, $t = + 3/5$
4. $6x^2 + x - 1 = 0$	$x = -\frac{1}{2}$, $x = 1/3$
5. $7x^2 - 9x - 10 = 0$	$x = 2$, $x = -5/7$
6. $3y^2 - 7y + 4 = 0$	$y = 4/3$, $y = 1$
7. $5x^2 + 7x - 6 = 0$	$x = -2$, $x = 3/5$
8. $4x^2 + 9x + 3 = 0$	$x = \frac{-9 + \sqrt{33}}{8}$ $x = \frac{-9 - \sqrt{33}}{8}$
9. $2s^2 + 2s - 1 = 0$	$s = \frac{-1 + \sqrt{3}}{2}$ $s = \frac{-1 - \sqrt{3}}{2}$
10. $12x^2 - 28x + 15 = 0$	$x = 5/6$, $x = 3/2$
11. $8x^2 + 4x - 1$	$x = \frac{-1 + \sqrt{3}}{4}$ $x = \frac{-1 - \sqrt{3}}{4}$
12. $12v^2 - 7v + 1 = 0$	$v = 1/4$, $v = 1/3$
13. $6x^2 - 3x - 2 = 0$	$x = \frac{3 + \sqrt{57}}{12}$ $x = \frac{3 - \sqrt{57}}{12}$
14. $15x^2 + 29x - 14 = 0$	$x = 2/5$, $x = -7/3$
15. $aw^2 + bw + c = 0$	$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

B. SOLVE THE FOLLOWING BY THE QUADRATIC FORMULA.	ANSWER
1. $x^2 + 8x + 3 = 0$	$x = - 4 - 2\sqrt{13}$, $x = - 4 + 2\sqrt{13}$
2. $6x^2 - 5x + 1 = 0$	$x = \frac{1}{2}$, $x = 1/3$
3. $25x^2 - 5x - 6 = 0$	$x = -2/5$, $x = 3/5$
4. $2w^2 - 5w - 3 = 0$	$w = 3$, $w = -\frac{1}{2}$
5. $2x^2 - 6x - 3 = 0$	$x = \frac{3 + \sqrt{15}}{2}$, $x = \frac{3 - \sqrt{15}}{2}$

6. $a^2 + 3a + 1 = 0$	$a = \frac{-3 + \sqrt{5}}{2}$, $a = \frac{-3 - \sqrt{5}}{2}$
7. $6x^2 + 3x - 1 = 0$	$x = \frac{-3 \pm \sqrt{33}}{12}$
8. $2x^2 + 7x + 2 = 0$	$x = \frac{-7 + \sqrt{33}}{4}$, $x = \frac{-7 - \sqrt{33}}{4}$
9. $4x^2 - 3x - 1 = 0$	$x = 1$, $x = -\frac{1}{4}$
10. $3a^2 - 4 = 0$	$a = \frac{2\sqrt{3}}{3}$, $a = \frac{-2\sqrt{3}}{3}$

UNIT 20

Supplementary Drill

Frames 6932-7127

A. WHICH OF THE FOLLOWING ARE TRUE PROPORTIONS:	ANSWER
1. $\frac{3}{8} = \frac{6}{16}$	true
2. $\frac{5}{7} = \frac{10}{14}$	true
3. $\frac{3}{2} = \frac{5}{4}$	false
4. $\frac{7}{12} = \frac{49}{144}$	false
5. $\frac{1}{5} = \frac{4}{20}$	true
6. $\frac{7}{8} = \frac{8}{7}$	false
7. $\frac{4}{7} = \frac{16}{28}$	true
8. $\frac{15}{3} = \frac{75}{15}$	true
9. $\frac{5}{8} = \frac{40}{64}$	true
10. $\frac{9}{7} = \frac{54}{49}$	false

B. SOLVE:	ANSWER
1. $\frac{a}{5} = \frac{3}{15}$	$a = 1$
2. $\frac{x}{12} = \frac{36}{72}$	$x = 6$
3. $\frac{9}{7} = \frac{s}{21}$	$s = 27$
4. $\frac{18}{32} = \frac{9}{y}$	$y = 16$
5. What number is to 15 as 3 is to 5?	9
6. What number is to 40 as 3 is to 10?	12
7. Six is to 8 as what number is to 24?	18

8. Four is to what number as 32 is to 48? 6

9. If it takes 1 quart of oil to travel 600 miles, how many quarts are required to travel 3600 miles? 6 qts.

10. A man 5 feet 6 inches tall casts a shadow of 8 feet. How long is the shadow of a tree 18 feet in height? (nearest tenth of a foot.) 26 2/11 ft.

11. If 5 baseballs cost $14.50, how much does it cost for 30 baseballs? $87.

12. A boy can paint 300 square feet of surface in $1\frac{1}{2}$ hours. How long will it take him to paint 1350 square feet? 6.75 hours

13. Two boys are sitting on a plank. If the boy weighing 130 lbs. is sitting 6 ft. from the fulcrum, how far must the boy weighing 78 lbs. sit from the fulcrum in order to balance? 10 ft.

14. A lever is 18 feet long. How far must a 3 lb. and 24 lb. weight be placed from the fulcrum in order to balance? 16 ft., 2 ft.

15. The sum of the digits of a number is 8. The tens digit is 4 greater than the units digit. What is the number? 62

16. The sum of the two digits of a number is 7. If the digits are interchanged the resulting number is 27 larger than the original number. Find the number. 25

17. A two digit number is 1 more than 6 times the sum of the digits. If the tens digit is 1 greater than the units digit, find the number? 43

18. Jim had $4.60. If he had as many quarters as he did halves, twice as many dimes as he had quarters, and twice as many nickels as he had dimes, how many of each kind did he have? 4 quarters, 4 halves, 8 dimes, 16 nickels

19. A man receives 93 cents in change. He has 2 more quarters than dimes, and 5 more pennies than quarters. How many of each coin did he have? 8 pennies
1 dime, 3 quarters

20. Three hundred tickets were sold for a play. Some cost 25 cents and some cost 10 cents. If $49.20 was the total box office take, how many of each price ticket did they sell?
10-cent 172
25-cent 128

UNIT 21

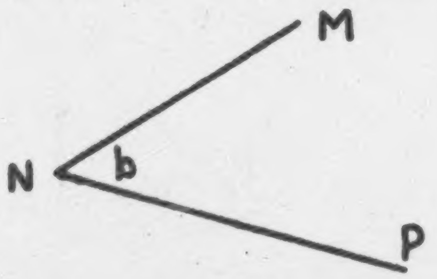
Supplementary Drill

Frames 7128-7308

A. WHAT TYPE OF VARIATION IS ILLUSTRATED BY THE FOLLOWING EXAMPLES. x and y are variables, K is a constant.

		ANSWER
1.	$x = Ky$	directly
2.	$K = x/y$	directly
3.	$xy = K$	inversely
4.	$x = 16/y$	inversely
5.	$y = 7/x$	inversely
6.	$x/y = 8$	directly
7.	$y = x/K$	directly
8.	$xy = 22$	inversely
9.	$x = 3K/y$	inversely
10.	$x = 4y$	directly

B. NAME THE ANGLE IN FOUR DIFFERENT WAYS:



ANSWER

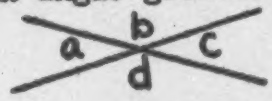
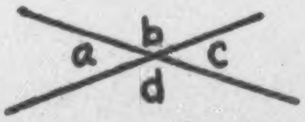
Angle b

Angle N

Angle MNP

Angle PNM

C. MATCH THE SECOND LIST WITH THE FIRST:

		ANSWER
1. obtuse	a) the supplement of a 95° angle.	h
2. vertical angles	b) An angle greater than 0° and less than 90°.	c
3. 67½°	c)  a and c are	i
4. 45°	d)  a and b are	j
5. acute	e) the complement of a 25° angle.	b
6. 65°	f) an angle containing 180°	e
7. right angle	g) an angle containing 90°	g

8. supplementary	h) an angle greater than 90° and less than 180.	d
9. straight angle	i) an angle which is three times as great as its complement.	f
10. 85°	j) an angle whose supplement is three times its complement.	a

UNIT 22

Supplementary Drill **Frames 7309-7551**

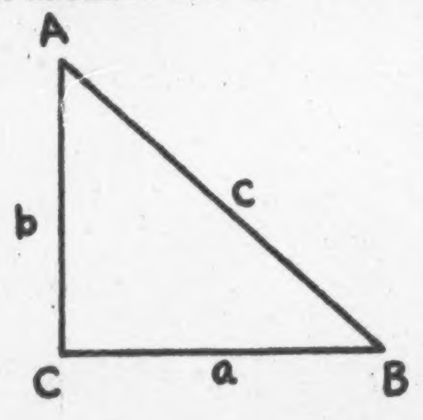
A. FIND THE MISSING SIDE IN EACH OF THE FOLLOWING RIGHT TRIANGLES: (leave square roots in radical form)

	Leg 1	Leg 2	Hypotenuse	Answer
1.	6	?	10	8
2.	5	12	?	13
3.	?	8	12	$4\sqrt{5}$
4.	12	?	22	$2\sqrt{35}$
5.	6	10	?	$2\sqrt{34}$

1. The sides of a triangle are 6, 7, and 8. Find the shortest side of a similar triangle if the longest side is 14. 21/2

2. The perimeter of a triangle is 48 inches. If the sides of this triangle are in the ratio of 3: 4: 5, what is the perimeter of a similar triangle whose shortest side is 8 inches? 32 inches

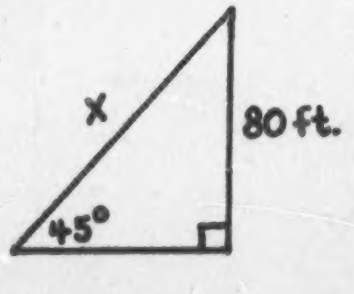
B. GIVEN THE FOLLOWING RIGHT TRIANGLE. WRITE THE THREE TRIGOMETRIC FUNCTION FOR ANGLE A AND B.



	ANSWER
sin ∠A =	a/c
cos ∠A =	b/c
tan ∠A =	a/b
sin ∠B =	b/c
cos ∠B =	a/c
tan ∠B =	b/a

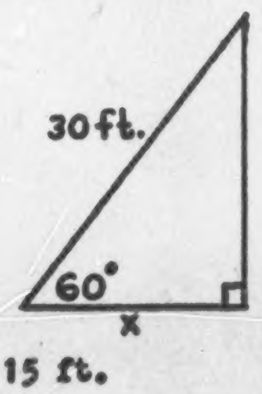
C. USING THE TABLE OF TRIGOMETRIC FUNCTIONS IN THE SUPPLEMENT, SOLVE THE FOLLOWING PROBLEMS FOR x:

1.



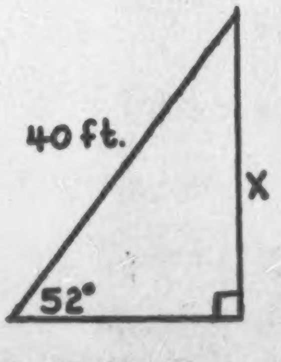
x = 113.16 ft.

2.



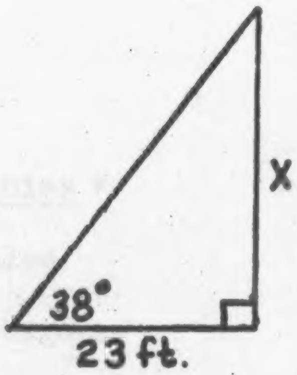
x = 15 ft.

3.



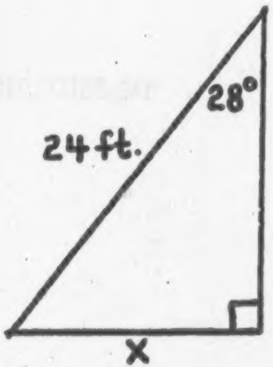
x = 31.52 ft.

4.



x = 17.96 ft.

5.



x = 11.26 ft.

SUPPLEMENTARY ENRICHMENT

Biographies *

1. Thales
2. Pythagoras
3. Zeno
4. Aristotle
5. Euclid
6. Eratosthenes
7. Archimedes
8. Hypatia
9. Fibonacci
10. Tartaglia
11. Cardano
12. Fermat
13. Banneker
14. Descartes
15. Pascal
16. Newton
17. Leibniz
18. Euler
19. Gauss
20. Abel
21. Boole
22. Poincaré
23. Einstein
24. Dedekind
25. Dase

* For source materials see Biography section of the Bibliography.

Selected Topics

* References correspond to numbered bibliography.

1. Abacus

   * History - 2, 4. Special Topics - 2, 41, 63.

2. Boolean Algebra

   Modern Mathematics - 3, 19, 29.

3. Calculating Prodigies

   Special Topics - 5, 6, 7, 54, 62. Mathematical Recreations - 3.

4. Calculating Rapidly

   Special Topics - 29(7), 65.

5. Casting Out Nines

   Mathematical Recreations - 1, 13.

6. Computers

   Special Topics - 13, 14, 15, 23, 51(1), 72, 76(1).

7. Fermat's Last Theorem

   History - 3. Special Topics - 26. Mathematical Recreations - 3.

8. Fibonacci Numbers

   History - 3. Special Topics - 9, 16, 51(14). Mathematical Recreations -13.

9. Fields (Algebraic)

   Modern Mathematics - 19, 21. Special Topics - 39.

10. Four Color Problem

    Special Topics - 4, 27. Mathematical Recreations - 15.

11. Golden Section

    Traditional Mathematics - 14. Special Topics - 11.

12. Graphs

    Special Topics - 29(1), 50, 51(7).

13. Group Theory

Traditional Mathematics - 8. Special Topics - 11, 21.

14. Inequalities

Special Topics - 10, 18, 45.

15. Infinity

Popular Expositions - 12. Modern Mathematics - 11. Special Topics - 52, 76(2), 77.

16. Large Numbers Including Googol and Googolplex

Popular Expositions - 5. Special Topics - 20, 22.

17. Linear Programming

Modern Mathematics - 15. Special Topics - 40, 51(16).

18. Magic Squares

Mathematical Recreations - 3, 10, 13.

19. Mathematical Fallacies

Popular Expositions - 5. Mathematical Recreations - 3.

20. Möbius Strips

Special Topics - 31, 71.

21. Napier Rods (Bones)

History - 2, 3.

22. Nim - A Binary Game

Traditional Mathematics - 8. Mathematical Recreations - 3, 7.

23. Number Systems

Popular Expositions - 2. History - 3. Special Topics - 29(12).
Mathematical Recreations - 1, 13.

24. Number Tricks

Popular Expositions - 2. Mathematical Recreations - 1, 4, 22.

25. Paper Folding (Origami).

Mathematical Recreations - 11, 18, 24.

26. Pi

Special Topics - 8, 44, 57.

27. Prime Numbers

Special Topics - 42, 43, 47.

28. Pythagorean Theorem

History - 3. Special Topics - 12, 17, 29(8), 32.

29. Rhind Papyrus

History - 1, 3. Special Topics - 28.

30. Sets

Modern Mathematics - 7. Special Topics - 24, 29(6), 36, 46, 68, 73.

31. Topology - Its Unusual Applications

Popular Expositions - 5, 12. Special Topics - 11, 29(11), 70.

32. Towers of Hanoi and Chinese Rings

Popular Expositions - 5. Mathematical Recreations - 1, 3.

33. Truth Tables

Popular Expositions - 3. Traditional Mathematics - 8. Modern Mathematics - 20, 25.

34. Zeno's Paradoxes

History - 3. Biography - 3. Popular Expositions - 5. Mathematical Recreations - 15.

35. Zero - Its Invention and Usuage

Popular Expositions - 6. Special Topics - 11, 25, 52.

Suggested Activities

1. Invent new symbols and a new numeration system.
2. Construct and demonstrate how to use a transit.
3. Devise an original field, group, or ring.
4. Solve an original problem using linear programming
5. Invent a new system of measurement with appropriate units and measuring devices.
6. A study of ancient number systems.
7. Calculation of pi.
8. Investigate potential careers in mathematics.
9. A study of some elementary concepts of probability.
10. Write an original mathematical poem, essay, or story.

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Producers: Bucknell University, Lewisburg, Pa.
    Doubleday and Co., Inc., Garden City, Long Island, New York.
    Encyclopaedia Britannica Films, Inc., Wilmette, Ill., Attn: TEMAC
    New York University of Technology, New York, New York
    SRA Materials of instruction, Science Research Associates, Inc., 259 East Erie Street, Chicago, 11, Illinois.
    Teaching Machines, Inc., 235 San Pedra Drive, N.E. Albuquerque, New Mexico

Numbers	Subject	Producer	Type of Response		Branching	Correct Response-Advancement	Consumable Materials	Audio-Visual	Cost	No. of Frames	Frame Exposures Single	Available 9/1/1963	Type of Course		Grade Level
			Mult. Choice	Const.									Basic	Supplement	
1.	Algebra (Adventures In)	Doubleday Tutortext	X		X	X		X	3.95	350	X	X	X		8-10
2.	Algebra (Introduction To)	Doubleday Tutorfilm	X		X	X	Film	X	35.00	350	X	X			8 up
3.	Algebra I	Teaching Machines		X			X		7.50	2000	X	X	X	X	
4.	Algebra I	Program Teaching Aids		X	X						X	X	X		8-10
5.	Algebra I	E.B.F.		X		X			12.75 sp.50	8200	X	X	X		H.Sh.
6.	Slide Rule (use of)	Doubleday Tutortext	X		X	X		X	3.95	400	X	X	X	X	9 up
7.	Algebra	N. Y. Institute of Technology			X			X		10000		X			C
8.	Algebra	National Teaching Machines		X			X	X	5.50	2,000	X		X	X	C./Hs
9.	Algebraic Structures Groups and Fields	Bucknell		X		X				1,650		X	X	X	12
10.	Fields and Ordered Fields	E.B.F.		X		X			7.50	1,828	X	X	X		C
11.	Set, Relations and Functions	Bucknell		X		X				1,050			X	X	12
12.	Logarithms	New York Inst. of Technology			X			X		600		X			A
13.	Modern Mathematics	Science Research		X					10.00	8,400	X	X	X		9or8
14.	Modern Mathematics	Teaching Mach.		X			X				X	X	X	X	
15.	Computer Mathematics	Teaching Mach.		X			X				X	X	X	X	
16.	Binary Numbers	New York Inst. of Technology			X			X		600		X			A

FILM REFERENCES

ADDITION AND SUBTRACTION OF RATIONAL NUMBERS , Traditional, 31 minutes. Modern Learning Aids, Summit, New Jersey. Rental $6.00 each for 3 school days. Begins with a brief review of Film #3201, Natural Numbers, Integers and Rational Numbers, and leads into a discussion of the properties of rational numbers, emphasizing the conditions under which definitions are not valid. Includes magnitudes, negative numbers, the idea of the absolute value, and the sums, and differences of positive and negative numbers.

ADVENTURES IN NUMBER AND SPACE, Modern, Association Films, Inc., Ridgefield, New Jersey. Rental $7.50. Dr. Howard Fehr, and Bill Baird Marionettes.

ALGEBRA IN EVERYDAY LIFE, Traditional, 11 minutes. Coronet Films, Chicago 1, Illinois. Rental $7.00, Indiana University. Presents the nature of algebra and its practical applications in an everyday situation, thus motivating interest in its study.

ALGEBRAIC AND COMPLEX FRACTIONS, Traditional, 29 minutes. Modern Learning Aids, Summit, New Jersey. Rental $6.00 per 3 days. The significance of fractional notation for algebraic symbols is reviewed. The product of two fractions, the reciprocial of a reciprocal, and the invert-and multiply rule are all presented. The addition, and subtraction of algebraic fractions is shown by using a common denominator. Also simplifying.

BASE AND PLACE, Modern, 30 minutes. Association Film, Inc., Ridgefield, New Jersey. Rental $5.75 Pennsylvania State University Audio-Visual Library. Presents the characteristics, history, and applications of the binary system. (digital computers)

BIG NUMBERS, Traditional, 31 minutes. Association Films, Inc., LaGrange, Illinois. Rental $5.75 Pennsylvania State University Audio-Visual Liabrary. Demonstrates how scientists, and mathematicians write, and use very large, and very small numbers. Stress the meaning, and importance of exponents, and shows some of their uses.

CAREERS IN MATHEMATICS, Traditional, 30 minutes. Pennsylvania State University Audio-Visual Library. Rental $5.75. Recap of eight films in the series. Rewards of a career in mathematics.

* CONCEPT OF FUNCTION, Modern, 16 minutes. McGraw-Hill, New York. Dr. Albert E. Meder, Jr., narrator. Rental $4.75, Pennsylvania State University Audio-Visual Library. A new approach to the concept of "function" which eliminated the former ambiguity, and presents new and clearer definitions of such terms as: F(x), range domain, and rule. Confusion between sinle-valued, and multiple-valued functions is eliminated. * These films are available without cost from the Department of Public Instruction.

DEVELOPING AND SOLVING LINEAR EQUATIONS, Traditional, 30 minutes. Modern Learning Aids, Summit, New Jersey. Rental $6.00 per 3 days. The proposition is first made that x stands for a number, with no restrictions. Slowly the need for the establishment of certain consistent restrictions on x is shown. The general solution of any linear equation is discussed, and the importance of checking solutions is stressed.

DONALD IN MATH-MAGIC LAND, Traditional. Association Films, Inc., LaGrange, Illinois. (Walt Disney film)

THE EARLIEST NUMBERS, Traditional, 30 minutes. Association Film, Inc. Rental $5.75, Pennsylvania State University Audio-Visual Library. Stresses man's first efforts to count in terms of the essential elements of a modern numeration system.

FIVE FUNDAMENTAL POSTULATES FOR ALGEBRA, Modern, 30 minutes. Modern Learning Aids, Summit, New Jersey. Rental $6.00 per 3 days. Shows that the rules are rules and not tested theorems. Covers the five postolates. The meaning of division, and the exclusion of division by zero are covered. The addition of fractions, and the meaning of the least common multiple and the lowest common denominator are reviewed.

FUNDAMENTAL OPERATIONS, Modern, 30 minutes. Association Films, Inc. Rental $5.75, Pennsylvania State University Audio-Visual Library. Analyzes modular arithmetic. Demonstrates that the two fundamental operations are addition, and multiplication: that subtraction, and division are the inverse of these operations.

HISTORICAL INTRODUCTION TO ALGEBRA, Modern, 27 minutes. Modern Learning Aids. Rental $6.00 per 3 days. Growth of mathematical knowledge. Also questions dealing with recent times.

HOW MAN LEARNED TO COUNT, Traditional. Association Films, Inc. Rental $7.50. A trip back through history to see how arithmetic developed.

HOW TO FIND THE ANSWER, Traditional, 11 minutes. Coronet Films. Rental Indiana University. Stresses the importance of applying the correct mathematical procedure to an appropriate problem and shows how this is done.

INTRODUCTION TO FACTORING, Modern, 30 minutes. Modern Learning Aids. Rental $6.00. The concept of factoring algebraic expression is introduced. Methods used in arithmetic are applied to examples in algebra. Warns about concelling across add or subtract signs. Factoring forms are shown for Monomial common term, Difference of two squares, Quadratic Perfect square.

INTRODUCTION TO QUADRATIC EQUATIONS, Modern, 30 minutes. Modern Learning Aids. Rental $6.00 per 3 days. The solution of word problems leading to quadratic equations is worked out in detail, including the use of the method of completing the square. The Quadratic Formula is derived and used to solve an earlier problem discussed in the film.

* IRRATIONAL NUMBERS, Modern, 23 minutes. McGraw-Hill, 1959. Rental $4.75, Pennsylvania State University Audio-Visual Library. Dr. Albert E. Meder, narrator. A demonstration for teaching irrational numbers through the study of decimal fractions. The limited understanding in past teaching is eliminated by this new method which leads to a positive demonstration of what an irrational number is. * These films are available without cost from the Department of Public Instruction.

IT'S ALL ARRANGED, Traditional, 30 minutes. Pennsylvania State University Audio-Visual Library, rental $5.75. Bill Baird and his Marionettes figure combinations mathematically.

LANGUAGE OF GRAPHS, Traditional. Coronet Films. Rental Indiana University.

THE LANGUAGE OF MATHEMATICS, Traditional, 11 minutes. Coronet Films. Rental Indiana University. Shows that the precise language of mathematics is frequently essential to the variety of problems.

LINEAR EQUATIONS IN ONE UNKNOWN, Traditional, 30 minutes. Modern Learning Aids. Rental $6.00. Two word problems are solved in detail. The first is finding the "point of no return" in an airplane flight, and the second involves computing the amount of alcohol needed to produce a certain percentage of mixture in an automobile radiator. Film ends with a discusion of equivalent equations and extraneous roots.

MATHEMATICIAN AND THE RIVER, Traditional, 20 minutes. Pennsylvania State University Audio-Visual Library. Rental $7.00. Problems in applied science on the Mississippi River; shows the relationship between "abstract world of mathematics, and the real world of mature."

MEANING OF Pi, Traditional. Coronet Films. Rental Indiana University.

MULTIPLICATION OF RATIONAL NUMBERS, Traditional, 31 minutes. Modern Learning Aids. Completes the review of the rational system and the fundamental operations of algebra. The multiplication, and division of signed numbers is developed carefully. Applications of these concepts are carried through the addition, subtraction, multiplication, and division of multiple term expressions involving powers.

MYSTERIOUS "X", Traditional, 30 minutes. Association Films, 1958. Rental Pennsylvania State University Audio-Visual Library, $5.75. Everyday applications of algebra.

NATURAL NUMBERS, INTEGERS AND RATIONAL NUMBERS, Traditional, 29 minutes. Modern Learning Aids. A presentation of the fundamental properties of addition, and multiplication. The need for the concepts of zero, and of negative numbers leads to the system of integers, and the inclusion of reciprocals produces the system of rational numbers.

NEW NUMBERS, Modern, 30 minutes. Association Films, Inc. Rental $5.75 Pennsylvania State University Audio-Visual Library. Explains new and important number concepts in modern mathematics. Indicates the patern of how new numbers arise. (irrational, transfinite, and complex numbers)

* NUMBER FIELDS, Modern, 17 minutes. McGraw-Hill, Dr. Albert E. Meder, Jr., narrator. Rental Pennsylvania State University Audio-Visual Library. A discussion, with animated drawings of integers, real numbers, rational numbers and complex numbers from the point of view of number fields, explaining the requirements and properties of a number field.

* PATTERNS IN MATHEMATICS, Modern, 14 minutes. McGraw-Hill. Dr. Albert E. Meder, Jr., narrator. Rental Pennsylvania State University Audio-Visual Library $2.75. Dr. Meder explains clearly and with varied examples why mathematics should be a study of pattern, rather than a collection of isolated facts and figures.

PENNSYLVANIA DEPARTMENT OF PUBLIC INSTRUCTION, Traditional, or Modern. Algebraic Symbols and Equations in Problem Solving Number Patterns, Sets, and Sentences.

QUICKER THAN YOU THINK, Modern, 30 minutes. Association Films, 1958. Rental Pennsylvania State University Audio-Visual Library. $5.75. Evolution of counting into the modern electronic calculator.

RADICALS AND THE REAL NUMBER SYSTEM, Traditional, 29 minutes. Modern Learning Aids. Rental $6.00 per 3 days. Shows how a line segment can be constructed to represent the lengths of irrational numbers. The real Number System includes all such numbers as well as rational numbers. Radicals of negative numbers are not included in this system. These ideas are presented and examples of handling are given.

RATIO AND PROPORTION, Traditional, 11 minutes. Coronet Junior High. Rental Indiana University.

* SENTENCES AND SOLUTION SETS, Modern, 21 minutes. McGraw-Hill. Dr. Albert E. Meder, Jr., narrator. Rental $4.25 Pennsylvania State University Audio-Visual Library. An explanation of how to introduce sets, variables, and open sentences, using actual classroom scenes, showing how the concept of set makes the teaching of inequalities and equations simpler and clearer.

SETS, CROWS, AND INFINITY, Modern, 12 minutes. Film Association of California, Los Angeles. The film explores the mathematical ideas of sets and one-to-one correspondence between two sets. An infinite set is shown to be a set which is equivalent to the infinite set of whole numbers.

THE SLIDE RULE: "C" AND "D" SCALE, Traditional, 22 minutes. Pennsylvania State University Audio-Visual Library. Rental $4.75. THE SLIDE RULE: PROPORTION, PERCENTAGE, SQUARES, AND SQUARE ROOTS, 21 minutes. Rental $4.25.

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